Chapter 2. Probability

§2.1 Sample Spaces & Events (& Set Operations)

Experiment is an activity or process with multiple possible results

Sample space of experiment is set of **all** possible outcomes

An **event** is a collection of possible outcomes from the sample space (outcomes that we are interested in)

Example: Experiment is flipping a coin until you get "Heads" — count # of flips

Sample Space is $\{1, 2, 3, 4, \dots\}$

H TH TTH etc

Event for "< 5 flips" is

$\{1, 2, 3, 4\}$

H TH TTH TTTH

Event for "≥ 5 flips" is

$\{5, 6, 7, \dots\}$

Example: Experiment is rolling two 6-sided dice & checking rolls.

Sample Space is

D1	D2
1	1
1	2
1	3
⋮	

36 possible rolls

Suppose we want to beat opponent's roll, which added to 10.

⇒ Interested in

Event = {Sum is ≥ 10} =

D1	D2
5	6
6	5
6	6

Note: Sample space can be finite or infinite

An event is **simple** if it consists of exactly **one** outcome

An event is **compound** if it consists of **more than one** outcome

An event is **null** if it is empty "$\varnothing$" (consisting of **no** outcomes)

Example: Experiment is rolling two dice.

Event 1 $E_1 = \{\text{sum of dice is } 12\} = \{(6, 6)\}$ (D1, D2)

simple

Event 2 $E_2 = \{\text{sum of dice is } 11\} = \{(5, 6), (6, 5)\}$ (D1, D2)

compound

Event 3 $E_3 = \{\text{sum of dice is } 13\} = \varnothing$

null

For complicated events it is often easiest to describe them as combinations of simpler events

Example: Experiment is rolling two dice

$$\{\text{at least one die is } 6\} = \{\text{D1 is } 6 \text{ or D2 is } 6\}$$

("union")

$$= \{\text{D1 is } 6\} \cup \{\text{D2 is } 6\}$$

$$\{(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\} \cup \{(1,6)\ (2,6)\ (3,6)\ (4,6)\ (5,6)\ (6,6)\}$$

$$\{\text{sum of dice is } \le 11\} = \{\text{sum of dice is not } \ge 11\}$$

$$= \{\text{All outcomes except for sum of dice is } \ge 11\}$$

$$= \{\text{sum of dice is } \ge 11\}'$$ ("complement")

Set theory operations:

Union $A \cup B$ "A or B"

↑ all outcomes in A or in B (or in both)

Intersection $A \cap B$ "A and B"

↑ only outcomes that are in both A & B

Complement A' (or A^c) "not A"

↑ all outcomes except for those in A

If $A \cap B = \varnothing$ (no outcomes are in both)

then we say A & B are "mutually exclusive" or "disjoint"